4
One-dimensional consolidation

In the previous chapter we studied briefly the conditions of the steady flow of water through a stationary soil structure; in this chapter we shall be concerned with the more general conditions of *transient* flow of water from and through soil-material. The seepage forces experienced by the soil structure will vary with time, and the soil structure itself may deform under the varying loads it sustains.

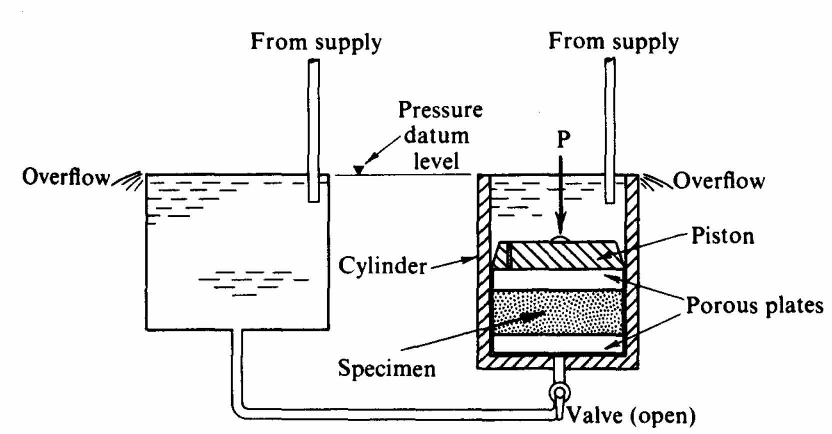

Fig. 4.1 Consolidometer

While the soil structure will exhibit compressibility, its response to an external compressive load will take time – the time necessary for the transient flow of pore-water to cease as the excess pore- pressures dissipate.

The time-dependent process during which a soil specimen responds to compression is commonly called the *process of consolidation.* Laboratory tests for measuring such compressibility of a soil are conducted either in a consolidometer (sometimes known as an oedometer) or in an axial compression cell (usually known as a 'triaxial' cell). We shall here confine our attention to a consolidometer, the principles of which are illustrated in Fig. 4.1. The apparatus consists essentially of a rigid metal cylinder with closed base, containing a soil sample in the shape of a thick circular plate sandwiched between two thin porous plates. The porous plates, generally made of ceramic or some suitable stone, allow the passage of water into or (more usually) out of the soil sample, but their pores are not sufficiently large to allow any of the soil particles to pass through them. Both porous plates are connected to constant head reservoirs of water, and vertical loads are applied to the sample by means of a closely fitting piston. A test consists of the instantaneous application of a constant increment of load and observation of the consequent settlement of the piston (and hence consolidation of the sample) with time.

4.1 Spring Analogy

The soil sample in the consolidometer is at all times carrying the total vertical load transmitted by the piston; and, as such, forms a load-carrying system. But the soil is a two-phase continuum, and it contains two separate materials (water and the structure of soil particles) which can be thought of as two independent structural members in parallel, as, for example, two members in parallel could make a double strut. The two members have markedly different stress – strain characteristics, and we can only discover the share of the total load taken by each (at any one instant) by considering both their statical equilibrium *and* strain compatibility, just as we have to do in the example of a composite member such as a double strut. We shall continue to assume that the water is *incompressible* compared with the soil structure. (Typical values are compressibility of water = 48×10^{-6} cm^2/kg and compressibility of London clay = 3000×10^{-6} cm^2/kg.)

There is an analogy between the behaviour of the soil sample in the consolidometer, and that of the spring in Fig. 4.2. (Note our caveat that an isotropic elastic continuum cannot properly be represented by a system of springs.) The spring is also in a consolidometer, fully surrounded by water and supporting a similar piston, except that it is assumed to be frictionless, weightless, and leakproof. The lead connecting the bottom of the cylinder to the reservoir above the piston contains a valve and is of small bore so that the water can only flow very slowly between the cylinder and reservoir, with a resulting pressure difference between the two containers.

Let us carry out a test in parallel on both soil sample and spring by applying a total vertical load *P* to both pistons. This additional load *P* will act at all levels within

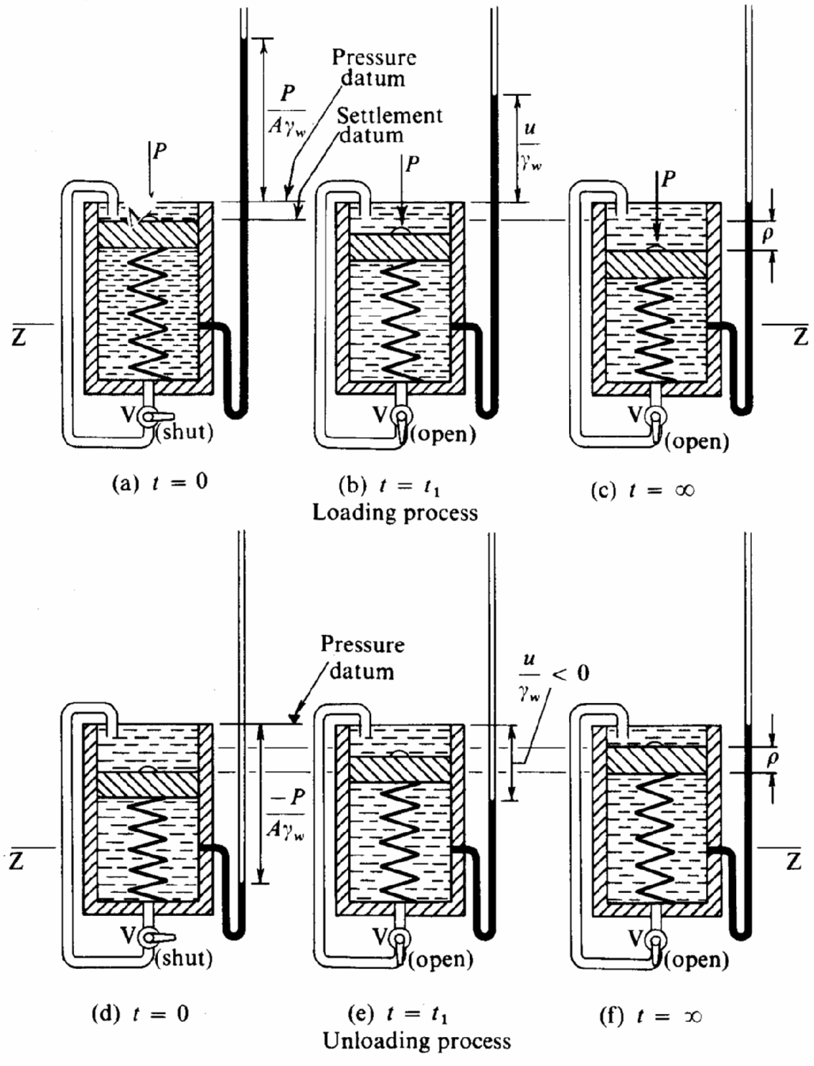

Fig. 4.2 Spring Analogy of Consolidation Process

the cylinder, but the proportions carried by the two structural members (water and soil) will vary not only with time but also with the particular level being considered. At any instant and at a certain level **ZZ** let the vertical load carried by the soil or spring be P_s, and that by the water P_w; then simple statics demands that at all times $P_s+P_w=P$. The load carried by the water is measured as excess pore-pressure *u,* recorded by the pressure tapping and manometer tube shown in Fig. 4.2.

If we divide these forces by the total cross-sectional area *A* of the cylinder (as for seepage) we have

$$\sigma' = {P_s}/{A} = effective\ stress$$

$$u = {P_w}/{A} = excess\ pore-pressure\ (neglecting\ the\ self\ weight\ of\ the\ soil\ and\ piston)$$

$$\sigma = (\sigma' + u) = {P}/{A} = total\ applied\ stress$$

Loading process

Before the start of the test the valve *V* is closed so that immediately the load *P* is applied, no water can escape from the cylinder, the piston cannot move, the soil or spring cannot be strained and cannot therefore experience any stress. Hence, at the very start, when $t = 0, \sigma' = 0, u = \sigma = P/A$ and we have the case of Fig. 4.2(a).

We now open the valve and observe that the water flows out of the cylinder into the reservoir, that the piston settles, and by so doing starts to compress the spring. At time $t = t_1$ after the valve is opened we have the situation of Fig. 4.2(b).

$0 < \sigma' = (\sigma - u)$; σ' increasing with time

$\sigma > u > 0$; u decreasing with time.

The rate of flow of water, and hence of settlement, will decay with time, and eventually after infinite time all excess pore-pressure will have dissipated, all settlement ceased, and all the load will be carried by the spring or soil. We have then at $t = \infty$ the case of Fig. 4.2(c)

$$\sigma' = \sigma = \frac{P}{A}\ and\ u = 0$$

and a total settlement of the piston recorded as ρ.

Unloading process

We now attempt to reverse the process by removing the total applied load *P.* Immediately after having done this, no time has elapsed for water to be sucked into the cylinder and for the piston to rise, so the spring will still be compressed by p and exerting an upward force *P.* Since the total load is zero, this force must be matched by a 'negative' one provided by 'tension' or a drop in pressure in the water. Consequently, at *t* = 0 we shall observe Fig. 4.2(d)

$$\sigma' = P/A;\ u = -\sigma';\ \sigma = (\sigma' + u) = 0.$$

As time elapses, the spring stretches and pushes the (weightless) piston upwards as water is sucked into the cylinder from the reservoir. At time $t = t_1$ after the start of this unloading process we have the situation of Fig. 4.2(e)

$P/A > \sigma'$ decreasing with time

$-P/A < u(= -\sigma') < 0$; u increasing with time.

Finally, after infinite time the system will have reached equilibrium with $\sigma' = 0 = u$, and the piston will have regained its original position having risen by ρ, see Fig. 4.2(f). We shall find in the next section that the soil sample often does not unload with the same characteristics as for the loading process, and that not all the settlement ρ may be recovered.

4.2 Equilibrium States

If we carry out a consolidation test on a real soil, and observe the settlement of the piston as a function of time we shall obtain results which when plotted give a curve of the form of Fig. 4.3, that at first sight appears to have a 'horizontal' asymptote and to reach a maximum settlement which remains fixed at some point such as E.

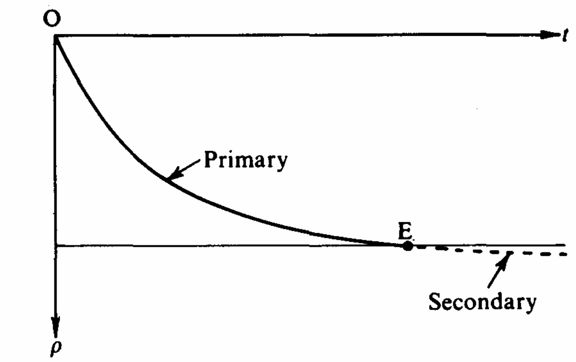

Fig. 4.3 Settlement of Piston

A second, closer look will show that after E is reached the piston will continue to settle indefinitely by a very small amount exhibiting the well-known phenomenon of creep. The settlement from O to E is known as *primary consolidation* and that after E as *secondary consolidation;* the choice of the position of E is not entirely arbitrary and a suitable definition will be given in §4.5. Throughout this chapter we shall be concerned only with primary consolidation, totally ignoring all secondary consolidation, so that we can consider E to represent effectively an *equilibrium state* marking the end of settlement under a given loading increment.

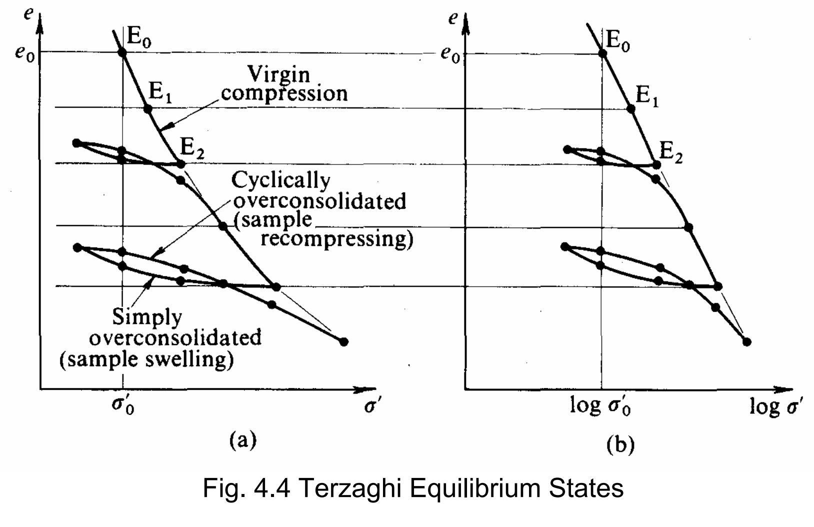

Fig. 4.4 Terzaghi Equilibrium States

If a series of load-increments is applied to a specimen which is initially very wet, the resulting equilibrium states will appear when plotted, as in Fig. 4.4. In Fig. 4.4(a) with the effective vertical stress σ' to a natural scale are the results of a sequence of loading and unloading processes, showing hysteresis loops where the specimen has been allowed to swell and then be reconsolidated. The outermost curve connecting the points when the specimen is in the loosest possible states is often called the virgin consolidation line, but will be referred to here as the *virgin compression line,* because it is strictly the line connecting equilibrium states whereas consolidation is a term used only for the time-dependent process between any pair of equilibrium states.

In Fig. 4.4(b), the same results are plotted with σ' on a logarithmic scale in which the virgin compression line becomes straight. Terzaghi1 fitted such data by a line of equation

$$e = e_0 - C_c \log_{10} \frac{\sigma'}{\sigma'_0} \tag{4.1}$$

in which the constant C_c, is called the compression index; it should be noted that the base 10 is used in this definition rather than the natural logarithm, which we denote by *ln*.

Later in this book instead of Terzaghi's expression, eq. (4.1), the virgin compression line will be denoted by the equation

$$v = v_0 - \lambda \ln \frac{p}{p_0}$$

and each swelling and recompression loop will be represented by a single straight line of the form

$$v = v_0 - \kappa \ln \frac{p}{p_0},$$

where λ and κ are characteristic constants for the soil. We suppose that the virgin compression λ-line represents an *irreversible* process, whereas the swelling and recompression κ-lines represent *reversible* processes.

4.3 Rate of Settlement

The rate of settlement of the piston that will occur inside the consolidometer will be governed by the speed with which water can be expelled from the innermost pores of the soil specimen: it will depend both on the permeability and on the length of drainage path. In the case of sand, the permeability is so high that the process of consolidation is effectively instantaneous, whereas for a clay layer in the ground, settlement may continue for years. The following analysis is relevant to clays.

In Fig. *4.5* we consider a suitably thin horizontal layer of the soil specimen and investigate its condition at some instant during the consolidation process during virgin compression. We assume that the process is entirely one-dimensional (with no lateral variations) so that the velocity and pressure of the pore-water are only functions of the depth *z* measured vertically downwards from the surface of the specimen, and of the time *t*. We imagine that two small probes are inserted so that the pore-pressures of the top and bottom surfaces of the layer are known. Our sign convention will require that the artificial velocity v_a is taken as positive *downwards* in the direction of *z,* and opposite to that of the permeameter in Fig. 3.1; also a positive increment of excess head *h* is shown which will cause an upward velocity, i.e., negative v_a.

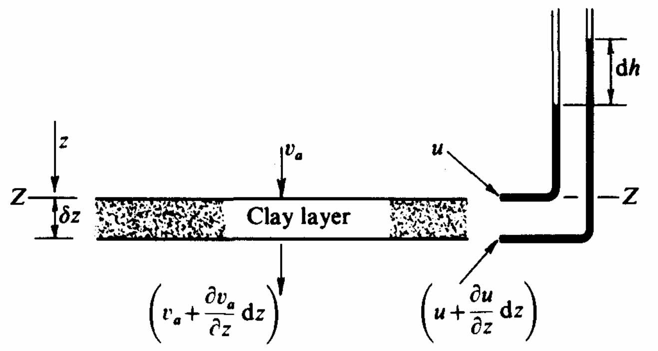

Fig. 4.5 Specimen Layer of Clay

The loading increment $\dot{\sigma}(=\delta\sigma)$ applied to the specimen is such that the initial and final equilibrium states are E_0 and E_1, shown in Fig. 4.6. At the present time *t* after the start of the consolidation process, the state of the thin layer will be represented by some point T between E_0 and E_1. The conventional assumption introduced by Terzaghi in his classic theory* of primary consolidation is that T always lies on the straight line joining E_0 and E_1. The slope of this straight line, which will change for different initial states E_0, is defined as

$$-\frac{\mathrm{d}e}{\mathrm{d}\sigma'}=(1+e_0)m_{vc} \tag{4.2}$$

where m_{vc} is the coefficient of volume compressibility. A similar constant m_{vs} but of much smaller value will be required for steps in the swelling and recompression loops of Fig. 4.4.

This definition allows us to relate settlement with change in effective stress. Let δp be the settlement experienced by the thin layer during time δt. The reduction in volume of the layer will be $A\ \delta p$ which can be expressed as

$$A\,\delta z\left(-\frac{\delta e}{1+e_0}\right),$$

by reference to Fig. 1.4

$$\therefore \qquad \frac{\delta p}{\delta z}=-\frac{\delta e}{1+e_0}=m_{vc}\delta\sigma'. \tag{4.3}$$

Because of continuity, the reduction in volume of the thin layer must be exactly matched by the volume of water expelled, which during time δt will be $(\partial v_a/\partial z)A\,\delta z\,\delta t$. Hence, in the limit we have

$$\frac{\partial v_a}{\partial z}=\frac{-1}{1+e_0}\frac{\partial e}{\partial t}=m_{vc}\frac{\partial\sigma'}{\partial t}. \tag{4.4}$$

Since the loading increment is a *fixed* one of $\dot{\sigma}=\delta\sigma$ we have $\delta\sigma'=\delta u=\sigma'=$ constant so that $\partial\sigma'/\partial t=-\partial u/\partial t$, and eq. (4.4) can be taken a step further to give

$$\frac{\partial v_a}{\partial z}=-m_{vc}\frac{\partial u}{\partial t}. \tag{4.5}$$

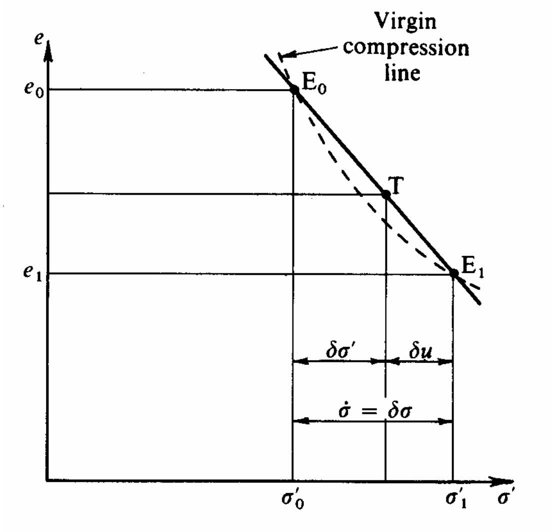

Fig. 4.6 Loading Increment

* We continue to use the parameter *e* here as in eq. (4.1) although in subsequent chapters we will use exclusively the parameter $v = 1 + e$.

Finally, we can obtain another relationship between v_a and u by employing Darcy's law,

$$v_a = ki = -k\frac{dh}{ds} = \frac{-k}{\gamma_w}\frac{\partial u}{\partial z}. \tag{4.6}$$

Differentiating this with respect to *z,* and combining with eq. (4.5)

$$\frac{+k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = -\frac{\partial v_a}{\partial z} = +m_{vc}\frac{\partial u}{\partial t}.$$

This is Terzaghi's fundamental differential equation for one- dimensional consolidation, usually written as

$$\frac{\partial u}{\partial t} = c_{vc}\frac{\partial^2 u}{\partial z^2} \tag{4.7}$$

where $c_{vc} = k/\gamma_w m_{vc}$ is the coefficient of consolidation; a typical value for a stiff clay is 2×10^{-3} in.^{2}/min. Once having solved the consolidation equation for a particular set of boundary conditions, we can calculate for any given time the total settlement ρ, the excess pore-pressure *u,* and effective stress σ' at any depth. In the next two sections an approximate and an exact solution will be derived for the one-dimensional consolidation of a clay layer under the same conditions as occur in the consolidometer.

4.4 Approximate Solution for Consolidometer

We have made an approximation to the physical problem in the construction of the model for one-dimensional consolidation. Before we become involved in Fourier's analysis in *§4.5,* we set out in this section an approximate solution[2] which fits the boundary conditions, but only has terms in z and z^2 so that it does not satisfy exactly the (already approximate) differential equation (4.7). We believe this solution to be helpful in providing an intuitive understanding of the various aspects of the problem.

Figure 4.7 represents part of a clay layer of large area A and depth H resting on an *impermeable* stratum so that no drainage can occur at the base: the boundary conditions are identical with that of the consolidometer when drainage is prevented at the base by closing the lead to the reservoir (Fig. 4.1). If we place a series of imaginary probes along the line AB at 45° to the horizontal to act as standpipes, their levels at any instant will provide a distribution of the excess pore-pressure u with depth z.

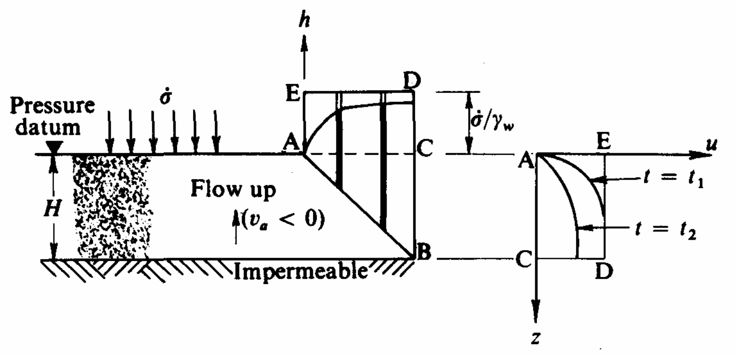

Fig. 4.7 Case of Consolidometer with no Drainage at Base

Immediately after the application of the increment of vertical load $\dot{\sigma}$ at t =0, the excess pore-pressure everywhere will be $\dot{\sigma}$ and all standpipes will show a level of $\dot{\sigma}/\gamma_w$ above pressure datum at the top of the clay layer. As time elapses, the excess pore-pressures dissipate as load is transferred to the soil structure and a series of lines

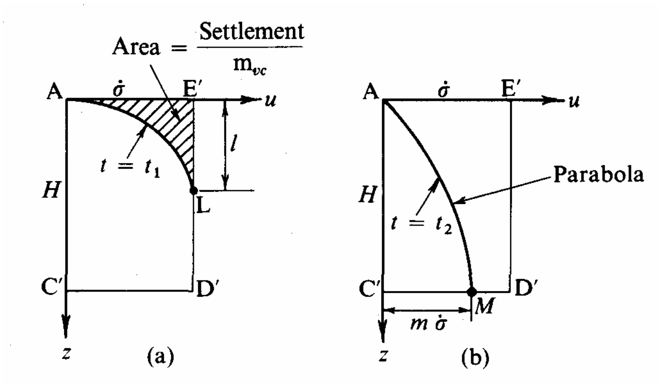

Fig. 4.8 Parabolic Isochrones

at $t = t_1$, $t = t_2$, etc., will be obtained. Each one is known as an *isochrone* (from the Greek 'equal time **'),** and since the length AC is equal to the depth H the gradient of the isochrone at any point is equal to d*h*/d*z* and represents the hydraulic gradient *i* that is acting upwards. In this case *i* is positive upwards and causing the flow of water to be vertically upwards everywhere in the clay layer. It is usual and convenient to depict the isochrones all rotated clockwise through 90° as seen in the right-hand part of Fig. 4.7.

We can obtain an approximate but fairly accurate solution by assuming that the isochrones are all parabolas and distinguishing two stages of consolidation as shown in Fig. 4.8. Immediately consolidation starts the pore-pressure at the surface A must drop to zero, and a typical isochrone for the first stage will be as AL in Fig. 4.8(a). Since the hydraulic gradient must be continuous (this is not a necessary condition in a layered soil) the parabola must have its apex at L with a vertical tangent there. The apex L rapidly moves down $E'D'$ and on reaching D' the first stage is complete.

During the second stage, a typical isochrone will appear as AM. This must have its apex at M with a vertical tangent there, because the boundary conditions at the base $D'C'$ demand no flow and hence zero hydraulic gradient. The point M gradually traverses $D'C'$ and eventually reaches C' when all consolidation is complete.

First stage

The isochrone AL for $t = t_1$ is given by

$$u = \dot{\sigma}\left(2\frac{z}{l} - \frac{z^2}{l^2}\right), \qquad 0 \le z \le l(t). \tag{4.8}$$

The settlement that has occurred will be, from eq. (4.3)

$$\rho_{t_1} = \int_0^l m_{vc}\sigma' \mathrm{d}z = \int_0^l m_{vc}(\dot{\sigma} - u)\mathrm{d}z$$

which is m_{vc} times the shaded area ALE'. This can readily be calculated to be $\frac{1}{3}m_{vc}\dot{\sigma}l$; so that the rate of settlement will be $\frac{1}{3}m_{vc}\dot{\sigma}\frac{\mathrm{d}l}{\mathrm{d}t}$, which must be equal (and opposite) to the rate of flow of water out of the surface. But this is the value of v_a for $z = 0$, i.e., from eq. (4.8) the upwards velocity is

$$(-v_a)_{z=0} = (-ki)_{z=0} = \frac{k}{\gamma_w}\left(\frac{\partial u}{\partial z}\right)_{z=0} = \frac{k\dot{\sigma}}{\gamma_w}\frac{2}{l}. \tag{4.9}$$

Hence

$$\frac{1}{3} m_{vc} \dot{\sigma} \frac{\mathrm{d} l}{\mathrm{d} t} = \frac{\mathrm{d}\rho}{\mathrm{d} t} = (-v_a)_{z=0} = \frac{k\dot{\sigma}}{\gamma_w} \frac{2}{l} \tag{4.10}$$

and integrating for the appropriate limits

$$l^2 = 12 c_{vc} t_1. \tag{4.11}$$

It is convenient here to introduce two useful parameters. The first is the proportion of the total settlement that has occurred so far, defined by

$$U = \frac{\rho_t}{\rho_\infty}. \tag{4.12}$$

The second is a dimensionless time factor defined by

$$T_v = \frac{c_{vc} t}{H^2}. \tag{4.13}$$

During this first stage of consolidation,

$$U = \frac{\rho_t}{\rho_\infty} = \frac{\frac{1}{3} m_{vc} \dot{\sigma}\, l}{m_{vc} \dot{\sigma}\, H} = \frac{\frac{1}{3}\sqrt{12 c_{vc} t}}{H} = \sqrt{4T_v / 3} \tag{4.12}$$

and the first stage will end when *l=H,* i.e., $T_v = \frac{1}{12}$.

Second stage $\left(T_v \geq \frac{1}{12} \right)$

The isochrone AM for $t = t_2$ is given by

$$u = \dot{\sigma}\, m \left(2\frac{z}{H} - \frac{z^2}{H^2} \right),\quad 1 \geq m(t) \geq 0. \tag{4.15}$$

Proceeding exactly similarly as for the first stage we have

$$\rho_{t_2} = m_{vc} \dot{\sigma}\, H \left(1 - \frac{2m}{3} \right) \tag{4.16}$$

$$(-v_a)_{z=0} = \frac{k\dot{\sigma}}{\gamma_w} \frac{2m}{H} \tag{4.17}$$

leading to

$$m_{vc} \dot{\sigma} H \left(-\frac{2}{3} \frac{\mathrm{d} m}{\mathrm{d} t} \right) = \frac{\mathrm{d}\rho}{\mathrm{d} t} = (-v_a)_{z=0} = \frac{k\dot{\sigma}}{\gamma_w} \frac{2m}{H} \tag{4.18}$$

and after integration

$$m = \exp\left(\frac{1}{4} - 3T_v \right). \tag{4.19}$$

The proportion of settlement

$$U = \frac{m_{vc} \dot{\sigma}\, H \left(1 - \frac{2m}{3} \right)}{m_{vc} \dot{\sigma}\, H} = 1 - \frac{2}{3} \exp\left(\frac{1}{4} - 3T_v \right). \tag{4.20}$$

If we now put the two stages of consolidation together and plot U against T_v we obtain Fig. 4.9 where in the first stage we have a straight line of slope $2/\sqrt{3}$ and the second a curve decaying exponentially.

Should the clay layer rest on a permeable stratum, as in Fig. 4.10, and drainage occur from both top and bottom surfaces, as will be the more usual case in laboratory experiments in the consolidometer, we can make immediate use of the above solution for the case of single drainage. This case of double drainage has symmetry about the mid-plane with all flow being upwards above this, and all flow being downwards below it. Typical isochrones are as shown, and we see that the top *half* has identical conditions with that of the case of single drainage. Hence we adopt the solution for single drainage, and *double* the settlements. In particular, in order to make use of the time factor, we must evaluate H as *half* of the depth in Fig. 4.10, and we can avoid any ambiguity by redefining it as the maximum drainage path, i.e., the longest path that any water particle has to travel to be expelled.

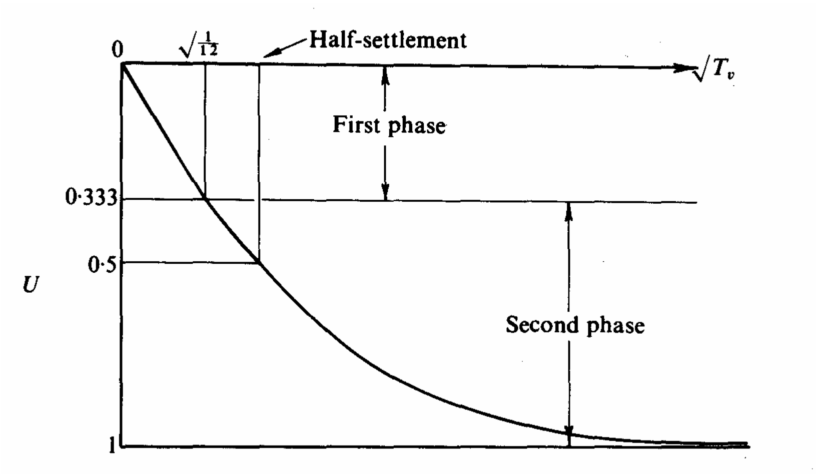

Fig. 4.9 Settlement Results from Parabolic Isochrones

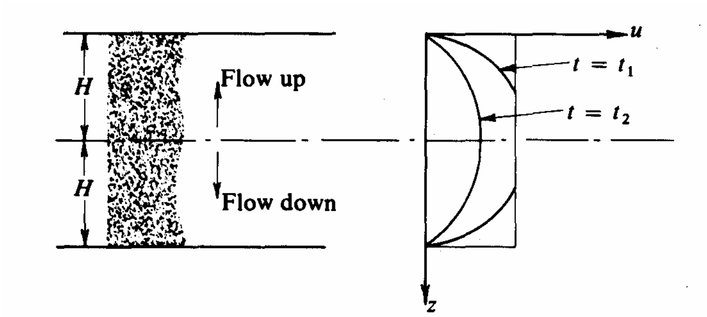

Fig. 4.10 Case of Consolidometer with Drainage at Both Surfaces

4.5 Exact Solution for Consolidometer

The above approximate solutions, although exactly fitting the boundary conditions, do not satisfy exactly the differential equation, (4.7). We can obtain an exact solution of the differential equation satisfying the relevant boundary conditions which for single drainage are

$$\left.\begin{array}{lll} t=0, & 0 \le z \le H, & u=\dot{\sigma} \\ 0<t\le\infty, & z=0, & u=0 \\ 0\le t\le\infty, & z=H, & v=-\dfrac{k}{\gamma_w}\dfrac{\partial u}{\partial z}=0 \\ t=\infty, & 0\le z\le H, & u=0. \end{array}\right\} \quad (4.21)$$

Our experience with the parabolic isochrones suggests that we separate variables and assume that u is the product of independent functions of z and t. We try $u = \phi(t)f(z)$ and substitution in eq. (4.7) then gives us

$$\frac{\phi'(t)}{\phi(t)} = c_{vc}\,\frac{f''(z)}{f(z)}. \tag{4.22}$$

Since this must hold for all z and all t, each side of this equation must be constant. For convenience let each side be equal to $-c_{vc}A^2$, and then by integrating we find

$$\left.\begin{aligned}\phi(t) &= \alpha\, e^{-A^2 c_{vc} t}\\ f(z) &= \beta \cos Az + \gamma \sin Az\end{aligned}\right\} \tag{4.23}$$

where α, β and γ are constants of integration. To satisfy the boundary conditions we might start by putting

$$u = \phi(t) f(z) = A' e^{-Mt} \sin\left(\frac{\pi z}{2H}\right)$$

where

$$Mt = \frac{\pi^2 c_{vc} t}{4H^2} = \frac{\pi^2 T_v}{4}$$

but this could never satisfy the first boundary condition as it is. So we have to employ Fourier analysis and take the infinite series

$$u = \sum_{m=1,2,\ldots}^{\infty} A_m e^{-m^2 Mt} \sin\left(\frac{m\pi z}{2H}\right)$$

requiring that

$$\dot{\sigma} = (u)_{t=0} = \sum A_m \sin\left(\frac{m\pi z}{2H}\right).$$

To solve this we adopt the standard procedure; we multiply by sin *(nπz/2H)* where *n* is an integer and integrate over a range of π to find

$$A_m = \frac{2\dot{\sigma}}{m\pi}(1 - \cos m\pi).$$

The complete solution then becomes

$$u = \sum_{m=1,2,\ldots}^{\infty} \frac{2\dot{\sigma}}{m\pi}(1 - \cos m\pi) \sin\left(\frac{m\pi z}{2H}\right) e^{-m^2 Mt} \tag{4.24}$$

and

$$U = 1 - \sum \frac{4}{m^2\pi^2}(1 - \cos m\pi) \sin\left(\frac{m\pi}{2}\right) e^{-m^2 Mt.} \tag{4.25}$$

Fortunately, because of the exponential factor both of these series are rapidly convergent and only two or three of their terms need be evaluated. For small values of t or T_v, the latter expression approximates to the straight line $U = \sqrt{4T_v / \pi}$ having a slope of 2/√π compared with 2/√3 for the parabolic isochrones.

Figure 4.11 shows the resulting curve obtained from eq. (4.25) compared both with experimental results obtained in the laboratory on specimens of saturated remoulded Gault Clay, and with the comparable approximate solution. The agreement is very close principally because with small thin laboratory specimens the assumptions and conditions implicit in Terzaghi's classic theory are fulfilled.

A method for establishing the end of primary consolidation suggested by Taylor[3] is as follows, as indicated in Fig. 4.11. The best straight line possible, E_0A, is drawn through the initial experimental points, and a second line, E_0B, with a slope 15 per cent less is drawn through the initial point, E_0. This line will cut the theoretical curve at U = 0.9, so that its intersection with the experimental curve is taken to be this point; hence the scale on the U-axis can be fixed together with the value of 100 per cent primary consolidation at E_1.

Further settlement after E_1, called secondary consolidation, is the topic of much current research.[4-6]

4.6 The Consolidation Problem

This chapter has considered transient flow of pore-water in one dimension, using the heat equation. The mathematically inclined reader may now expect us to generalize Terzaghi's model in some manner that leads us to an equation resembling that of three-dimensional heat transfer. Unfortunately, we cannot solve the consolidation problem with the same ease as was possible for the seepage problem. Terzaghi's model has a spring constant which is comparable to Young's modulus for an elastic body, and soil is no more capable of being reduced to a model with three orthogonal springs than is an isotropic elastic body. Terzaghi's model has one spring constant for virgin compression and a different constant for swelling, so we have to model a material with an elastic/plastic hardening character; we have to model the behaviour of the soil structure both in compression and in distortion; and if we can meet these difficulties we will be so far from the linear heat-transfer model that we shall not be able to make use of the elegant solution of problems of transient heat-transfer in three dimensions. However, we can make some general comments on the problem of transient flow in three dimensions by reference to what we now know of one-dimensional transient flow.

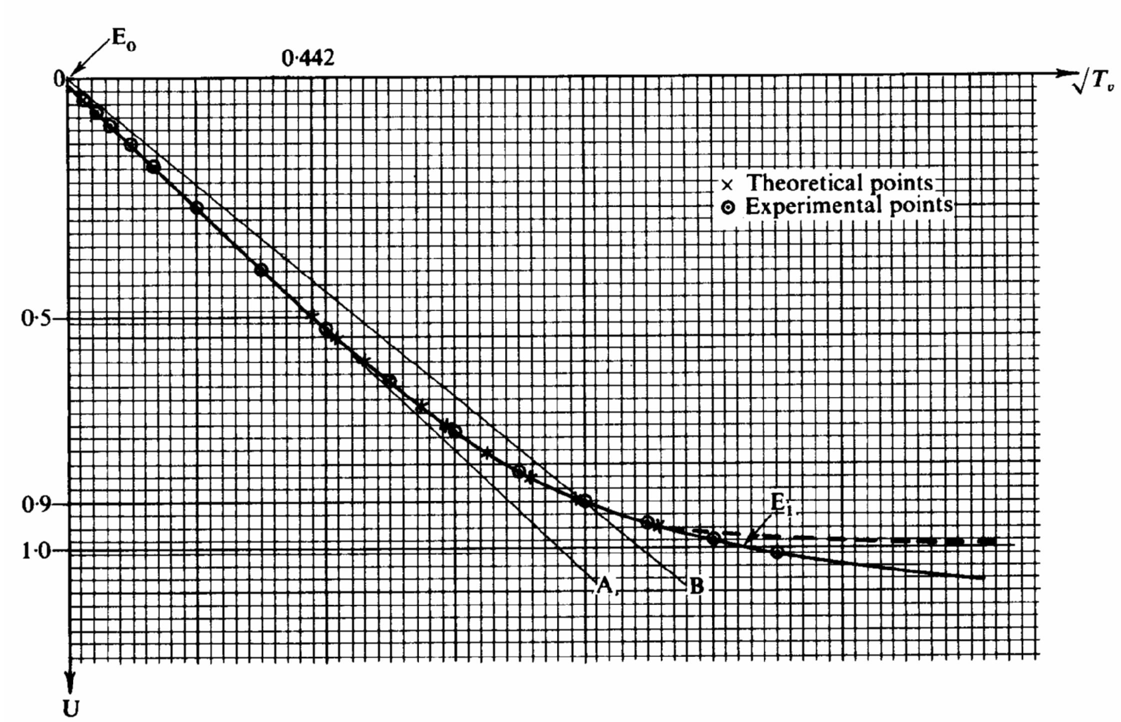

Fig. 4.11 Experimental Laboratory Results

In the one-dimensional problem the relative displacement of the two opposite surfaces (ends) of the body was called 'settlement' and equalled the total quantity of drainage out of the body per unit cross-sectional area. In general, an engineer will want to know a 'differential settlement', that is the relative displacement of two adjacent points of a three-dimensional body such as one point at the centre of an oil tank and another point at the edge of the tank. In this general case, the normal component of the rate of displacement of a point on the surface of the loaded soil body will not equal the rate of flow of seepage water through the surface at that point, and the rate of change of differential settlement of two points need not equal the rate of total flow of water out of the body.

Although an engineer might like to categorize any consolidation problem .by numerical values of two parameters:

(a) a maximum differential settlement, and

(b) the time required before halfsettlement* occurs,

the exact calculation of these two parameters will clearly be a formidable problem in any non-trivial case. However, approximate estimates may be made with less difficulty.

Let us calculate the half-settlement time factor $T_{1/2}$ for the problem of one-dimensional consolidation. We had an approximate solution using parabolic isochrones in eq. (4.20), into which we can introduce *U = 1/2* to give our first estimate

$$T_{1/2} = \frac{1}{3}\left(\frac{1}{4} - \ln\frac{3}{4}\right) = 0.18. \qquad (4.26)$$

The curve of *U* against $\sqrt{T_v}$ for this solution has an initial slope

$$\frac{U}{\sqrt{T_v}} = \frac{2}{\sqrt{3}}. \qquad (4.27)$$

We also had an exact series solution in eq. (4.25): the curve of *U* against $\sqrt{T_v}$ for that solution has a comparable initial slope

$$\frac{U}{\sqrt{T_v}} = \frac{2}{\sqrt{\pi}}, \qquad (4.28)$$

and gives an exact half-settlement time factor

$$T_{1/2} = 0.20 \qquad (4.29)$$

which is only 10 per cent different from the value estimated by the approximate solution.

For further comparison let us note results from exact solutions of two different problems. In the first case consider a layer where initial excess pore-pressure decreases linearly from some value at the drained surface to zero at $z = H$; this sort of distribution is generated when a vertical load is applied somewhere on the horizontal surface of a clay layer, and in known[1] solutions of this problem the half-settlement time factor is found to be

$$T_{1/2} \cong 0.09. \qquad (4.30)$$

In contrast, consider a second case of a layer where initial excess pore-pressure increases linearly from zero at the drained surface to some value at depth; this sort of distribution is generated when a body of soil is placed as a slurry in 'hydraulic fill', and for this case the half-settlement time factor is known[1] to be

$$T_{1/2} \cong 0.29. \qquad (4.30)$$

Comparison of these three values of $T_{1/2}$ suggests that a useful range of problems will have

$$0.1 < T_{1/2} < 0.3 \qquad (4.31)$$

which is a much smaller range of variation than is likely in H^2 and in c_{vc}.

We may generally calculate the time required before half-settlement occurs as

$$t_{1/2} = \frac{H^2 T_{1/2}}{c_{vc}}, \qquad (4.32)$$

and for a useful range of problems a crude numerical estimate can be made using $T_{1/2} = 0.2$.

* We have chosen the half-factor arbitrarily: some practicing engineers regard 80 per cent as significant. In limiting equilibrium calculations dissipation of 90 per cent of maximum excess pore-pressures is considered significant. Although these differences are important we will not discuss them here.

If we accept as a proper approximation that

$$t_{1/2} = 0.2\frac{H^2}{c_{vc}} \tag{4.34}$$

then we can draw a useful distinction between *undrained* and *drained* problems. For any given soil body let us suppose we know a certain time t_l in which a proposed load will gradually be brought to bear on the body; construction of an embankment might take a time t_l of several years, whereas filling of an oil tank might involve a time t_l, of less than a day. Then we can distinguish *undrained* problems as having $t_{1/2} >> t_l$, and *drained* problems as having $t << t_l$.

Equation (4.34) can be written

$$t_{1/2} = \frac{0.2\gamma_w H^2 m_{vc}}{k} \tag{4.35}$$

in which form we can see directly the effects of the different parameters m_{vc}, k, and H. More compressible bodies of virgin compressed soft soil will have values of m_{vc} associated with λ and longer half-settlement times than less compressible bodies of overcompressed firm soil having values of associated with k (elastic swelling or recompression). Less permeable clays will have much smaller values of k and hence longer half-settlement times than more permeable sands. Large homogeneous bodies of plastically deforming soft clay will have long drainage paths H comparable to the dimension of the body itself, whereas thin layers of soft clay within a rubble of firm clay will have lengths H comparable to the thinness of the soft clay layer. This distinction between those soils in which the undrained problem is likely to arise and those in which the drained problem is likely to arise will be of great importance later in chapter 8.

The engineer can control consolidation in various ways. The soil body can be pierced with sand-drains that reduce the half-settlement time. The half-settlement time may be left unaltered and construction work may be phased so that loads that are rather insensitive to settlement, such as layers of fill in an embankment, are placed in an early stage of consolidation and finishing works that are sensitive to settlement are left until a later stage; observation of settlement and of gradual dissipation of pore-pressure can be used to control such operations. Another approach is to design a flexible structure in which heavy loads are free to settle relative to lighter loads, or the engineer may prefer to underpin a structure and repair damage if and when it occurs. A different principle can be introduced in 'pre-loading' ground when a heavy pre-load is brought on to the ground, and after the early stage of consolidation it is replaced by a lighter working load: in this operation there is more than one ultimate differential settlement to consider.

In practice undetected layers of silt[6], or a highly anisotropic permeability, can completely alter the half-settlement time. Initial 'elastic' settlement or swelling can be an important part of actual differential settlements; previous secondary consolidation[7], or the pore-pressures associated with shear distortion may also have to be taken into account. Apart from these uncertainties the engineer faces many technical problems in observation of pore-pressures, and in sampling soil to obtain values of c_{vc}. While engineers are generally agreed on the great value of Terzaghi 's model of one- dimensional consolidation, and are agreed on the importance of observation of pore-pressures and settlements, this is the present limit of general agreement. In our opinion there must be considerable progress with the problems of quasi-static soil deformation before the general consolidation problem, with general transient flow and general soil deformation, can be discussed. We will now turn to consider some new models that describe soil deformation.

References to Chapter 4

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