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Basic concepts

1.1 Introduction

This book is about conceptual models that represent the mechanical behaviour of saturated remoulded soil. Each model involves a set of mechanical properties and each can be manipulated by techniques of applied mathematics familiar to engineers. The models represent, more or less accurately, several technically important aspects of the mechanical behaviour of the soil-material. The soil-material is considered to be a homogeneous mechanical mixture of two phases: one phase represents the structure of solid particles in the soil aggregate and the other phase represents the fluid water in the pores or voids of the aggregate. It is more difficult to understand this soil-material than the mechanically simple perfectly elastic or plastic materials, so most of the book is concerned with the mechanical interaction of the phases and the stress – strain properties of the soil-material in bulk. Much of this work is of interest to workers in other fields, but as we are civil engineers we will take particular interest in the standard tests and calculations of soil mechanics and foundation engineering.

It is appropriate at the outset of this book to comment on present standard practice in soil engineering. Most engineers in practice make calculations and base their judgement on the model used two hundred years ago by C. A. Coulomb¹ in his classic analysis of the active and passive pressures of soil against a retaining wall. In that model soil-material (or rock) was considered to remain rigid until there was some surface through the body of soil-material on which the shear stress could overcome *cohesion* and *internal friction*, whereupon the soil-material would become divided into two rigid bodies that could slip relative to each other along that surface. Cohesion and internal friction are properties of that model, and in order to make calculations it is necessary for engineers to attribute specific numerical values to these properties in each specific body of soil. Soil is difficult to sample, it is seldom homogeneous and isotropic in practice, and engineers have to exercise a considerable measure of subjective judgement in attributing properties to soil.

In attempts over the last half-century to make such judgements more objective, many research workers have tested specimens of saturated remoulded soil. To aid practising engineers the successive publications that have resulted from this continuing research effort have reported findings in terms of the standard conceptual model of Coulomb. For example, typical papers have included discussion about the ‘strain to mobilize full friction’ or ‘the effect of drainage conditions on apparent cohesion’. Much of this research is well understood by engineers, who make good the evident inadequacy of their standard conceptual model by recalling from their experience a variety of cases, in each of which a different interpretation has had to be given to the standard properties.

Recently, various research workers have also been developing new conceptual models. In particular, at Cambridge over the past decade, the *critical state* concept (introduced in §1.8 and extensively discussed in and after chapter 5) has been worked into a variety of models which are now well developed and acceptable in the context of isotropic hardening elastic/plastic media. In our judgement a stage has now been reached at which engineers could benefit from use of the new conceptual models in practice.

We wish to emphasize that much of what we are going to write is already incorporated by engineers in their present judgements. The new conceptual models incorporate both the standard Coulomb model and the variations which are commonly considered in practice: the words cohesion and friction, compressibility and consolidation, drained and undrained will be used here as in practice. What is new is the inter-relation of

concepts, the capacity to create new types of calculation, and the unification of the bases for judgement.

In the next section we begin abruptly with the model for sedimentation, and then in §1.3 we introduce the empirical index tests and promise a fundamental interpretation for them (which appears in chapter 6). Mechanical grading and index properties form a soil classification in §1.4, and in §1.5 we define the water content and specific volume of soil specimens. Then in §1.6 we introduce the concept of effective stress and in §1.7 digress to distinguish between the complex mathematical consequences of a simple concept (which will apply to the solution of limiting stress distributions by the method of characteristics in chapter 9) and the simple mathematical consequences of a complex concept (which will apply to the critical states).

1.2 Sedimentation and Sieving in Determination of Particle Sizes

The first model to be considered is Stokes' sphere, falling steadily under gravity in a viscous fluid. Prandtl^{2,3} discusses this as an example of motion at very small Reynolds' number. The total force of resistance to motion is found from reasons of dynamical similarity to be proportional to the product (viscosity μ) \times (velocity v) \times (diameter d). Stokes' solution gives a force of resistance R which has a coefficient 3π as follows:

$$R = 3\pi\mu vd \quad (1.1)$$

The force must be in equilibrium with the buoyant weight of the sphere, so that

$$3\pi\mu vd = \frac{\pi}{6}(G_s - 1)\gamma_w d^3 \quad (1.2)$$

where $G_s\gamma_w$ is the weight of unit volume of the solid material of the sphere and γ_w is the weight of unit volume of water. Hence, if a single sphere is observed steadily falling through a distance z in a time t , it can be calculated to have a diameter

$$d = \left\{ \frac{18\mu}{(G_s - 1)\gamma_w} \frac{z}{t} \right\}^{1/2} \quad (1.3)$$

This calculation is only appropriate for small Reynolds' numbers: Krumbein and Pettijohn⁴ consider that the calculation gives close estimates of diameters of spherical quartz particles settling in water provided the particle diameters are less than 0.005 mm. They also suggest that the calculation continues to hold until particle diameters become less than 0.1 microns (0.0001 mm), despite the effect of Brownian movement at that diameter. This range of applicability between 0.05 mm and 0.0001 mm makes a technique called *sedimentary analysis* particularly useful in sorting the sizes of particles of silt and clay soils.

Typical particles of silt and clay are not smooth and spherical in shape, nor does each separate particle have the same value of G , as may be determined from the aggregated particles in bulk. So the sizes of irregularly shaped small particles are *defined* in terms of their settling velocities and we then use adjectives such as 'hydraulic' or 'equivalent' or 'sedimentation' diameter. This is one example of a conceptual model in that it gives a meaning to words in the language of our subject.

The standard⁵ experimental technique for quantitative determination of the small particle size distribution in a soil is sedimentation. The soil is pre-treated to break it up mechanically and to dissolve organic matter and calcium compounds that may cement particles together. Then, to counteract certain surface effects that may tend to make particles flocculate together, a dispersion agent is added. About 10 gm of solid particles are dispersed in half a litre of water, and the suspension is shaken up in a tube that is repeatedly inverted. The test begins with a clock being started at the same instant that the

tube is placed in position in a constant temperature bath. After certain specified periods a pipette is inserted to a depth $z = 100$ mm in the tube, a few millilitres of the suspension are withdrawn at that depth and are transferred into a drying bottle.

Let us suppose that originally a weight W of solid material was dispersed in 500 millilitres of water, and after time t at depth 100 mm a weight w of solid material is found in a volume V of suspension withdrawn by the pipette. If the suspension had been sampled immediately at $t = 0$ then the weight w would have been $WV/500$ but, as time passes, w will have decreased below this initial value. If all particles were of a single size, with effective diameter D , and if we calculate a time

$$t_D = \frac{18\mu z}{(G_s - 1)\gamma_w D^2} \quad (1.4)$$

then *before* time t_D the concentration of sediment at the sampling depth ($z = 100$ mm) would remain at its initial value $WV/500$: *at* time t_D the particles initially at the surface of the tube would sink past the depth $z = 100$ mm, and *thereafter*, as is shown in Fig. 1.1, there would be clear liquid at the depth z .

It is evident that if any particles of one specified size are present at a depth z they are present in their original concentration; (this is rather like dropping a length of chain on the ground: as the links fall they should preserve their original spacing between centres, and they would only bunch up when they strike the heap of chain on the ground). Therefore, in the general case when we analyse a dispersion of various particle sizes, if we wish to know what fraction by weight of the particles are of diameter less than D , we must arrange to sample at the appropriate time t_D . The ratio between the weight w withdrawn at that time, and the initial value

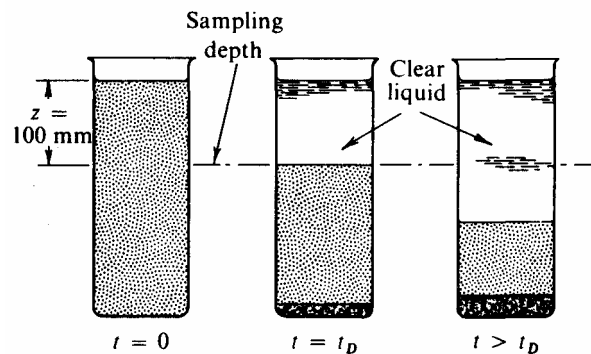


Fig. 1.1 Process of Sedimentation on Dispersed Specimen

$WV/500$, is the required fraction. The fraction is usually expressed as a percentage smaller than a certain size. The sizes are graded on a logarithmic scale. Values are usually found for $D = 0.06, 0.02, 0.006,$ and 0.002 mm, and these data⁶⁻¹¹ are plotted on a curve as shown in Fig. 1.2.

A different technique⁵, sieving, is used to sort out the sizes of soil particles of more than 0.2 mm diameter. A sieve is made with wire cloth (a mesh of two sets of wires woven together at right angles to each other). The apertures in this wire cloth will pass particles that have an appropriate intermediate or short diameter, provided that the sieve is shaken sufficiently for the particles to have a chance of approaching the holes in the right way. The finer sieves are specified by numbers, and for each number there is a standard¹² nominal size in the wire cloth. The coarser sieves are specified by nominal apertures. Clearly this technique implies a slightly different definition of diameter from that of sedimentary analysis, but a continuous line must be drawn across the particle size distribution chart of Fig. 1.2: the assumption is that the two definitions are equivalent in

the region about 0.05 mm diameter where sieves are almost too fine and sedimentary settlement almost too fast.

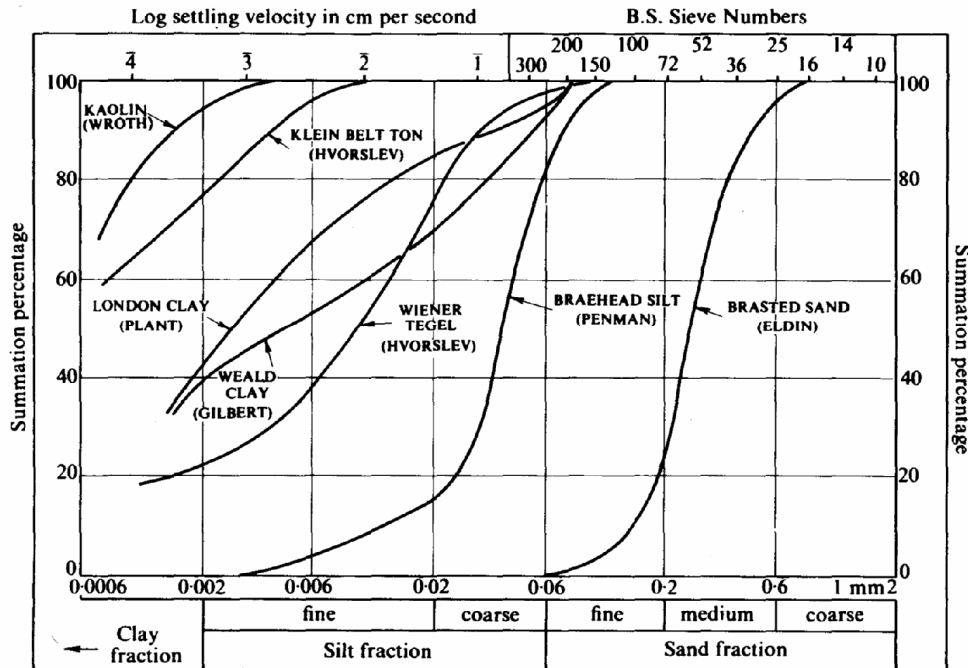


Fig. 1.2 Particle Size Distribution Curves

Civil engineers then use the classification for grain size devised at the Massachusetts Institute of Technology which defines the following names:

Boulders are particles coarser than 6 cm, or 60 mm diameter;

Gravel contains particles between 60 mm and 2 mm diameter;

Sand contains particles between 2 mm and 0.06 mm diameter;

Silt contains particles between 0.06 mm and 0.002 mm diameter;

Clay contains particles finer than 0.002 mm (called two microns, 2μ).

The boundaries between the particle sizes not only give almost equal spacing on the logarithmic scale for equivalent diameter in Fig. 1.2, but also correspond well with major changes in engineering properties. A variety of soils is displayed, including several that have been extensively tested with the results being discussed in detail in this text. For example, London clay has 43 per cent clay size, 51 per cent silt size and 6 per cent sand size.

1.3 Index Tests

The engineer relies chiefly on the mechanical grading of particle sizes in his description of soil but in addition, two index numbers are determined that describe the clayeyness of the finer fraction of soil. The soil is passed through a sieve, B.S. No. 36 or U.S. Standard No. 40, to remove coarse sand and gravel. In the first index test⁵ the finer fraction is remoulded into a paste with additional water in a shallow cup. As water is added the structure of fine soil particles is remoulded into looser states and so the paste becomes progressively less stiff. Eventually the soil paste has taken up sufficient water that it has the consistency of a thick cream; and then a groove in the paste (see Fig. 1.3) will close with the sides of the groove flowing together when the bottom of the cup is given a succession of 25 blows on its base. The paste is then at the 'lowest limit' of a continuing

range of liquid states: the water content (defined in §1.5) is determined and this is called the *liquid limit* (LL) of that soil.

The strength of the paste will increase if the paste is compressed either by externally applied pressure, or by the drying pressures that are induced as water evaporates away into the air and the contracting surfaces of water (menisci) make the soil particles shrink into more closely packed states. This hardening phenomenon will be discussed more extensively in later chapters; it will turn out (a) that the strength of the paste increases in direct proportion to the increase of pressure, and (b) that the reduction of water content is proportional to the logarithm of the ratio in which the pressure is increased.

In the second index test⁵ the soil paste is continuously remoulded and at the same time allowed to air-dry, until it is so stiff that when an attempt is made to deform the soil plastically into a thin thread of 1/8 in. diameter, the soil thread crumbles. At that state the soil has approximately a hundred times the strength that it had when remoulded at the higher water content that we call the liquid limit: because of the logarithmic relationship between pressure (or strength) and water content it will turn out that this hundred-fold increase in strength corresponds to a characteristic reduction of water content (dependent on the compressibility of the soil). The water content of the crumbled soil thread is determined and is called the *plastic limit* (PL). The reduction of water content from the liquid limit is calculated and is given the name *plasticity index* (PI) = (LL – PL). Soils with a high plasticity index are highly plastic and have a large range of water contents over which they can be remoulded: in English common speech they might be called ‘heavy’ clay soils.

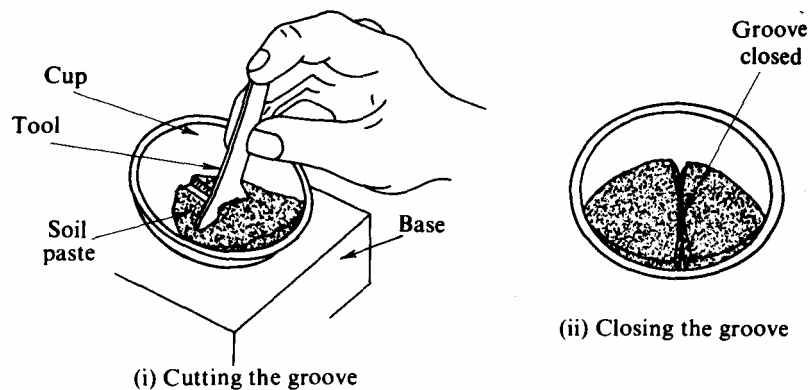


Fig. 1.3 Liquid Limit Test Groove (After Lambe²⁰)

Skempton¹³ found that there is a correlation between the plasticity index of a soil and the proportion of particles of clay size in the soil. If a given specimen of clay soil is mixed with various proportions of silt soil then there is a constant ratio of

$$\text{Activity} = \frac{\text{plasticity index}}{\text{percentage finer than two microns}}$$

The activity of clay soil depends on the clay minerals which form the solid phase and the solute ions in the water (or liquid phase).

1.4 Soil Classification

The engineer's classification of soil by mechanical grading and index tests may seem a little crude: there is a measure of subjective choice in the definition of a mechanical grading, the index tests at first appear rather arbitrary, and we have quite neglected to make any evaluations either of the fabric and origin of the soil, or of the nature of the clay minerals and their state in the clay-water system. However, in this section we will suggest

that the simple engineering classification does consider the most important mechanical attributes of soil.

It is hard to appreciate the significance of the immense diversity of sizes of soil particles. It may be helpful to imagine a city scene in which men are spreading tarred road stone in the pavement in front of a fifteen-story city building. If this scene were reduced in scale by a factor of two hundred thousand then a man of 1.8 metres in height would be nine microns high – the size of a medium silt particle; the building would be a third of a millimetre high – the size of a medium sand grain; the road stones would be a tenth of a micron – the size of what are called ‘colloidal’ particles; the layer of tar would correspond to a thickness of several water molecules around the colloidal particles. Our eyes could either focus on the tarred stones in a small area of road surface, or view the grouping of adjacent buildings as a whole; we could not see at one glance all the objects in that imaginary scene. The diversity of sizes of soil particles means that a complete survey of their geometry in a soil specimen is not feasible. If we select a volume of 1 m^3 of soil, large enough to contain one of the largest particles (a boulder) then this volume could also contain of the order of 10^8 sand grains and of the order of 10^{16} clay particles. A further problem in attempting such a geometrical or structural survey would be that the surface roughness of the large irregular particles would have to be defined with the same accuracy as the dimensions of the smallest particles.

An undisturbed soil can have a distinctive fabric. The various soil-forming processes may cause an ordering of constituents with concentration in some parts, and the creation of channels or voids in other parts. Evidence of these extensively occurring processes can be obtained by a study of the microstructure of the soil, and this can be useful in site investigation. The engineer does need to know what extent of any soil deposit in the field is represented by each specimen in the laboratory. Studies of the soil-forming processes, of the morphology of land forms, of the geological record of the site will be reflected in the words used in the description of the site investigation, but not in the estimates of mechanical strength of the various soils themselves.

In chapters 5 and 6 we consider macroscopically the mechanical strength of soil as a function of effective pressure and specific volume, without reference to any microscopic fabric. We will suggest that the major engineering attributes of real soils can be explained in terms of the mechanical properties of a homogeneous isotropic aggregate of soil particles and water. We show that the index properties are linked with the critical states of fully disordered soil, and we suggest that the critical state strengths form a proper basis of the stability of works currently designed by practising engineers.

Suppose we have a soil with a measured peak strength which (a) could not be correlated with index properties, (b) was destroyed after mechanical disturbance of the soil fabric, and (c) could only be explained in terms of this fabric. If we wished to base a design on this peak strength, special care would be needed to ensure that the whole deposit did have this particular (unstable) property. In contrast, if we can base a design on the macroscopic properties of soil in the critical states, we shall be concerned with more stable properties and we shall be able to make use of the data of a normal soil survey such as the *in situ* water content and index properties.

We gave the name clay to particles of less than 2 micron effective diameter. More properly the name clay should be reserved for clay minerals (kaolinite, montmorillonite, illite, etc.). Any substance when immersed in water will experience surface forces: when the substance is subdivided into small fragments the body forces diminish with the cube of size while surface forces diminish with the square of size, and when the fragments are less than 0.1 micron in size the substance is in ‘colloidal’ form where surface forces predominate. The hydrous-alumino-silicate clay minerals have a sheet-like molecular

structure with electric charge on the surfaces and edges. As a consequence clay mineral particles have additional capacity for ion-exchange. Clearly, a full description of the clay/water/solute system would require detailed studies of a physical and chemical type described in a standard text, such as that by Grim.¹⁴ However, the composite effects of these physico-chemical properties of remoulded clay are reflected to a large measure in the plasticity index. In §6.9 we show how variation of plasticity corresponds to variation in the critical states, and this approach can be developed as a possible explanation of phenomena such as the al sensitivity of leached post-glacial marine clay, observed by Bjerrum and Rosenqvist.¹⁵

In effect, when we reaffirm the standard soil engineering practice of regarding the *mechanical grading* and *index properties* as the basis of soil classification, we are asserting that the influence of mineralogy, chemistry and origin of a soil on its mechanical is behaviour is adequately measured by these simple index tests.

1.5 Water Content and Density of Saturated Soil Specimens

If a soil specimen is heated to 105°C most of the water is driven off, although a little will still remain in and around the clay minerals. Heating to a higher temperature would drive off some more water, but we stop at this arbitrary standard temperature. It is then supposed that the remaining volume of soil particles with the small amount of water they still hold is in effect ‘solid’ material, whereas all the water that has been evaporated is ‘liquid’.

This supposition makes a clear simplification of a complicated reality. Water at a greater distance from a clay particle has a higher energy and a lower density than water that has been adsorbed on the clay mineral surface. Water that wets a dry surface of a clay mineral particle emits ‘heat of wetting’ as the water molecules move in towards the surface; conversely drying requires heat transfer to remove water molecules off a wet surface. The engineering simplification bypasses this complicated problem of adsorption thermodynamics. Whatever remains after the sample has been dried at 105°C is called *solid*; the specific gravity (G_s) of this residue is found by experiment. Whatever evaporates when the sample is dried is called *pore-water* and it is assumed to have the specific gravity of pure water. From the weights of the sample before and after drying the water content is determined as the ratio:

$$\text{water content } w = \frac{\text{weight of pore water}}{\text{weight of solids}}.$$

In this book we will attach particular significance to the volume of space v occupied by unit volume of solids: we will call v the *specific volume* of unit volume of solids. Existing soil mechanics texts use an alternative symbol e called ‘voids ratio’ which is the ratio between the volume of ‘voids’ or pore space and the volume of solids: $v = 1 + e$. A further alternative symbol n called porosity is defined by $n = (v - 1) / v = e / v$. Figure 1.4(a) illustrates diagrammatically the unit volume of solids occupying a space v , and Fig. 1.4(b) shows separately the volumes and weights of the solids and the pore-water. In this book we will only consider *fully saturated soil*, with the space $(v - 1)$ full of pore-water.

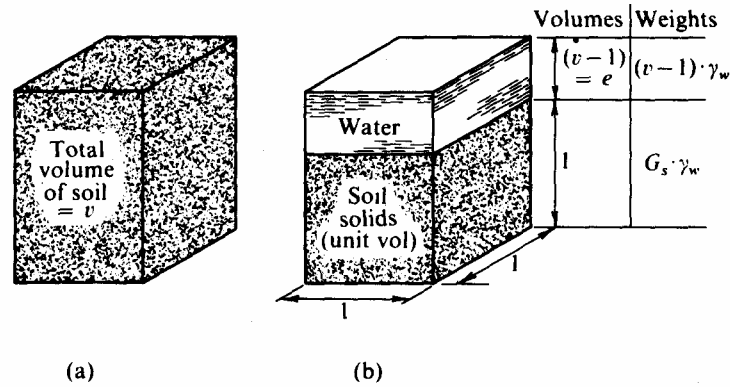


Fig. 1.4 Specific Volume of Saturated Soil

The total weight when saturated is $\{G_s + (v-1)\}\gamma_w$ and dry is $G_s\gamma_w$. These weights within a total volume v lead to the definition of

$$\text{saturated bulk density, } \gamma = \frac{G_s + v - 1}{v} \gamma_w = \frac{G_s + e}{1 + e} \gamma_w \quad (1.5)$$

$$\text{and dry bulk density, } \gamma_d = \frac{G_s}{v} \gamma_w.$$

It is useful to remember a relationship between the water content w and specific volume v of a saturated soil

$$v = 1 + G_s w. \quad (1.6)$$

Typical values are as follows: sand with $G_s = 2.65$ when in a *loose* state with $v = 1.8$ will have $\gamma = 1.92\gamma_w$; in a *dense* state with $v = 1.5$ will have $\gamma = 2.10\gamma_w$; clay with $G_s = 2.75$ when at the *liquid* limit might have $w = 0.7$ and then with $v = 1 + 2.75 \times 0.7 = 2.92$ it will have $\gamma = 1.60\gamma_w$; at the *plastic* limit it might have $w = 0.3$ and then with $v = 1 + 2.75 \times 0.3 = 1.83$ it will have $\gamma = 1.95\gamma_w$. Actual values in any real case will be determined by experiment.

1.6 The Effective Stress Concept

We will discuss stress and strain in more detail in the next chapter, but here we must introduce two topics: (a) the general the treatment of essentially discontinuous material as if it were continuous, and (b) the particular treatment of saturated soil as a two phase continuum.

The first of these topics has a long history.¹⁶ Navier treated elastic material as an assembly of molecular particles with systems of forces which were assumed to be proportional to the small changes of distance between particles. In his computation of the sum of forces on a particle Navier integrated over a sphere of action as if molecular forces were continuously distributed, and ended with a single elastic constant. Subsequently, in 1822 Cauchy replaced the notion of forces between molecular particles with the notion of distributed pressures on planes through the interior of a continuum. This work opened a century of successive developments in continuum mechanics. Cauchy and other early workers retained Navier's belief that the elasticity of a material could be characterized by a single constant but, when Green offered a derivation of the equations of elasticity based on a potential function rather than on a hypothesis about the molecular structure of the material, it became clear that the linear properties of an isotropic material had to be characterized by *two* constants – a bulk modulus and a shear modulus.

Thereafter, in the mathematical theory of elasticity and in other sections of continuum mechanics, success has attended studies which treat a volume of material

simply as a space in which some properties such as strain energy or plastic power are continuously distributed. So far the advances in solid state physics which have been accompanied by introduction of new materials and by new interpretations of the properties of known materials, have not led to a revival of Navier's formulation of elasticity. There is a clear distinction between workers in continuum mechanics who base solutions of boundary value problems on equations into which they introduce certain material constants determined by experiment, and workers in solid-state physics who discuss the material constants *per se*.

Words like specific volume, pressure, stress, strain are essential to a proper study of continuum mechanics. Once a sufficient set of these words is introduced all subsequent discussion is judged in the wider context of continuum mechanics, and no plea for special treatment of this or that material can be admitted. Compressed soil, and rolled steel, and nylon polymer, must have essentially equal status in continuum mechanics.

The particular treatment of saturated soil as a *two-phase* continuum, while perfectly proper in the context of continuum mechanics is sufficiently unusual to need comment. We have envisaged a distribution of clean solid particles in mechanical contact with each other, with water wetting everything except the most minute areas of interparticle contact, and with water filling every space not occupied by solids. The water is considered to be an incompressible liquid in which the *pore pressure* may vary from place to place. Pore-water may flow through the structure of particles under the influence of excess pore-pressures: if the structure of particles remains rigid a steady flow problem of *seepage* will arise, and if the structure of particles alters to a different density of packing the transient flow problem of *consolidation* will occur. The stress concept is discussed in chapter 2, and the total stress component σ normal to any plane in the soil is divided into two parts; the pore-pressure u_w and the *effective* stress component σ' , which must be considered to be effectively carried by the structure of soil particles. The pore-pressure u_w can be detected experimentally if a porous-tipped tube is inserted in the soil. The total stress component can be estimated from knowledge of the external forces and the weight of the soil body. The effective stress component σ' is simply calculated as

$$\sigma' = \sigma - u_w \quad (1.7)$$

and our basic supposition is that the mechanical behaviour of the effective soil structure depends on all the components of the *effective stress* and is quite independent of u_w .

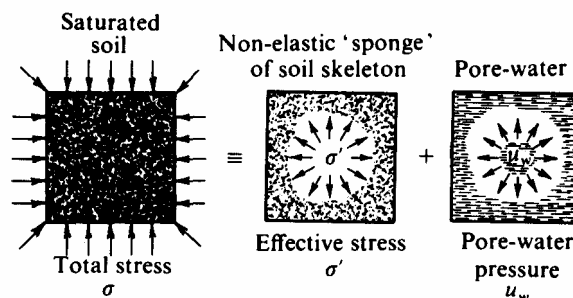


Fig. 1.5 Saturated Soil as Two-phase Continuum

In Fig. 1.5 the different phases are shown diagrammatically: each phase is assumed to occupy continuously the entire space, somewhat in the same manner that two vapours sharing a space are assumed to exert their own partial pressures. In Fig. 1.6 a simple tank is shown containing a layer of saturated soil on which is superimposed a layer of lead shot. The pore-pressure at the indicated depth in the layer is $u_w = \gamma_w \times h_w$, and this applies whether h_w is a metre of water in a laboratory or several kilometres of water in an ocean

abyss. The lead shot is applying the effective stress which controls the mechanical behaviour of the soil.

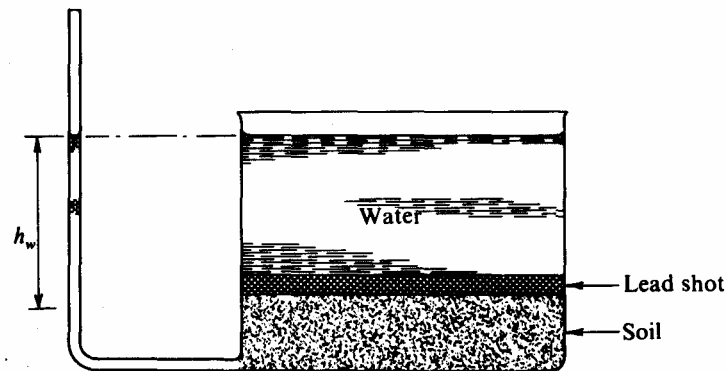


Fig. 1.6 Pore-pressure and Effective Stress

The introduction of this concept of effective stress by Terzaghi, and its subsequent generalization by Rendulic, was the essential first step in the development of a continuum theory of the mechanical behaviour of remoulded saturated soils.

1.7 Some Effects that are ‘Mathematical’ rather than ‘Physical’

Most texts on soil mechanics¹⁷ refer to work of Prandtl which solved the problem of plane punch indentation into perfectly *plastic* material; the same texts also refer to work of Boussinesq which solved various problems of contact stresses in a perfectly *elastic* material. The unprepared reader may be surprised by the contrast between these solutions. In the elastic case stress and strain vary continuously, and every load or boundary displacement causes some disturbance – albeit a subtle one – *everywhere* in the material. In direct contrast, the plastic case of limiting stress distribution leads to regions of *constant* stress set abruptly against fans of *varying* stress to form a crude patchwork.

We can begin to reconcile these differences when we realize that both types of solution must satisfy the same fundamental differential equations of equilibrium, but that the elastic equations are ‘elliptic’ in character¹⁸ whereas the plastic equations are ‘hyperbolic’. A rather similar situation occurs in compressible flow of gases, where two strongly contrasting physical regimes are the mathematical consequence of a *single* differential equation that comes from a single rather simple physical model.

The general differential equation of steady compressible flow in the (x, y) plane has the form¹⁹

$$(1 - M^2) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (1.8)$$

where ϕ is a potential function, and M (the Mach number) is the ratio u/v_{so} of the flow velocity u and the sonic velocity v_{so} corresponding to that flow. The model is simply one of a fluid with a limiting sonic velocity.

To illustrate this, we have an example in Fig. 1.7(a) of wavefronts emanating at speed v_{so} from a single fixed source of disturbance O in a fluid with uniform low-speed flow $u < v_{so}$. After a time t , the centre of the disturbance has been carried a distance ut downstream,

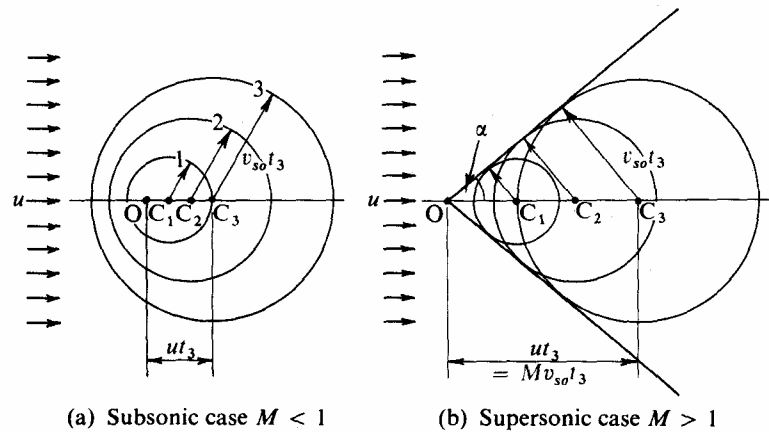


Fig. 1.7 Wavefronts Disturbing Uniform Flow

but this is less than the radius $v_{so}t$ of the wavefront. The wavefronts will eventually reach all points of the fluid.

In contrast, in Fig. 1.7(b) we have the same source disturbing a fluid with uniform high-speed flow $u > v_{so}$. In this case the wavefronts are carried so fast downstream that they can never cross the Mach cone (of angle 2α where $M \sin \alpha = 1$) and never reach points of the fluid outside it.

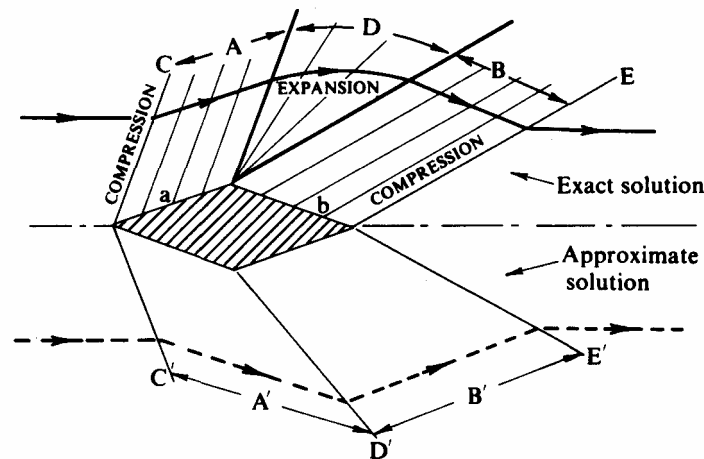


Fig. 1.8 High-speed Flow past an Aerofoil

In the subsonic case where $M < 1$ the general differential equation (1.8) is elliptic without real 'characteristic' directions (i.e., there is no real solution for α) and the Mach cone is not generated. In the supersonic case where $M > 1$, the general differential equation is hyperbolic with real 'characteristic' directions existing at angles $\pm \alpha$ to the direction of flow.

The example of high speed flow past an aerofoil is shown in Fig. 1.8; above the aerofoil we show an 'exact' pattern of characteristics and below the aerofoil an approximation. The directions of the characteristics are found from the local sonic velocity and the flow velocity. Conditions along straight segments 'a' and 'b' of the profile dictate straight supersonic flow in domains A and B. The flow lines have concentrated changes of direction at the compression shocks C and E; in the exact solution a steady change of direction occurs throughout the expansion wave D – a centred fan of characteristics – whereas in the approximation a concentrated expansion D' leads to a concentrated change

of direction. This approximation is valid for calculations of integrated effects near the aerofoil.

Some simple intuitive understanding of a problem of plane plastic flow can be gained from Fig. 1.9, which shows inverted extrusion of a contained metal billet pressed against a square die. The billet A is rigidly contained: the billet advances on the die with a certain velocity but when the metal reaches the fan B, it distorts in shear and it leaves the die as a rigid bar C with twice the billet velocity. Each element of metal is rigid until it enters the fan, it shears and flows within the fan, and on exit it abruptly freezes into the deformed pattern. The effect of the die is severe, and local. The metal in the fan is stressed to its yield limit and it is not possible to transmit more pressure from the billet to the die face through this yielding plastic material. In the mathematically comparable problems of supersonic flow we have regions into which it is not possible to propagate certain disturbances and the inability of the material to disperse these disturbances results in locally concentrated physical effects.

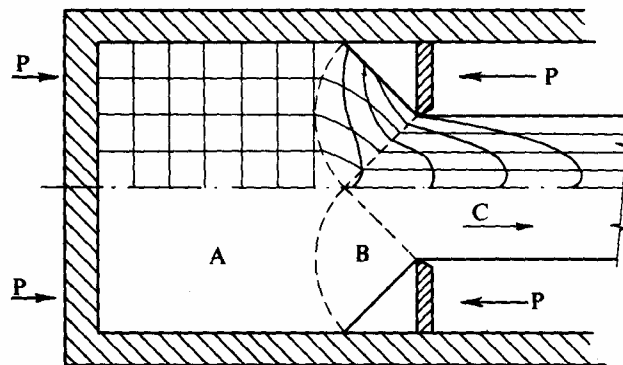


Fig. 1.9 Extrusion of Metal Billet (After Hill)

We have to explain in this book much more complicated models than the models of compressible flow or plastic shear flow. It will take us three more chapters to develop initial concepts of stress and strain and of steady and transient flow of water in soil. When we reach the new models for mechanical behaviour of a soil it will take four more chapters to explain their working, and the last chapter can only begin to review what is already known of the mathematical consequences of older models.

1.8 The Critical State Concept

The kernel of our ideas is the concept that soil and other granular materials, if continuously distorted until they flow as a frictional fluid, will come into a well-defined critical state determined by two equations

$$q = Mp$$

$$\Gamma = v + \lambda \ln p.$$

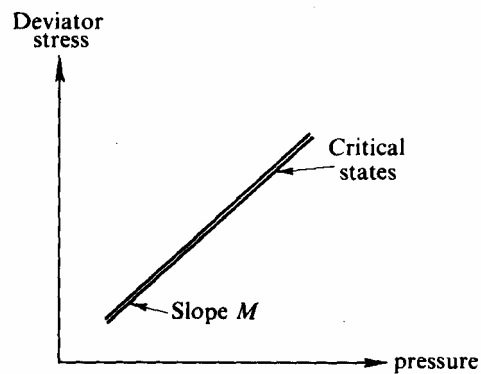
The constants M , Γ , and λ represent basic soil-material properties, and the parameters q , v , and p are defined in due course.

Consider a random aggregate of irregular 'solid' particles of diverse sizes which tear, rub, scratch, chip, and even bounce against each other during the process of continuous deformation. If the motion were viewed at close range we could see a stochastic process of random movements, but we keep our distance and see a continuous flow. At close range we would expect to find many complicated causes of power dissipation and some damage to particles; however, we stand back from the small details and loosely describe the whole process of power dissipation as 'friction', neglecting the

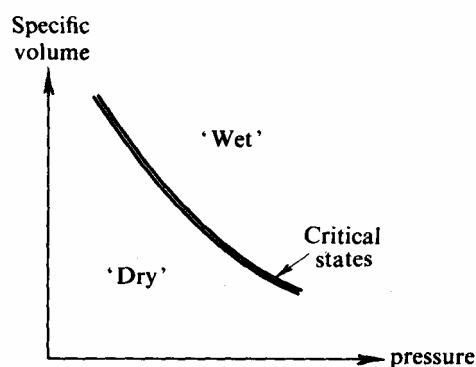
possibilities of degradation or of orientation of particles. The first equation of the critical states determines the magnitude of the 'deviator stress' q needed to keep the soil flowing continuously as the product of a frictional constant M with the effective pressure p , as illustrated in Fig. 1.10(a).

Microscopically, we would expect to find that when interparticle forces increased, the average distance between particle centres would decrease. Macroscopically, the second equation states that the specific volume v occupied by unit volume of flowing particles will decrease as the logarithm of the effective pressure increases (see Fig. 1.10(b)). Both these equations make sense for dry sand; they also make sense for saturated silty clay where low effective pressures result in large specific volumes - that is to say, more water in the voids and a clay paste of a softer consistency that flows under less deviator stress.

Specimens of remoulded soil can be obtained in very different states by different sequences of compression and unloading. Initial conditions are complicated, and it is a problem to decide how rigid a particular specimen will be and what will happen when it begins to yield. What we claim is that the problem is not so difficult if we consider the *ultimate* fully remoulded condition that might occur if the process of uniform distortion were carried on until the soil flowed as a frictional fluid. The *total change* from any initial state to an ultimate critical state can be precisely predicted, and our problem is reduced to calculating just how much of that total change can be expected when the distortion process is not carried too far.



(a)



(b)

Fig. 1.10 Critical States

The critical states become our base of reference. We combine the effective pressure and specific volume of soil in any state to plot a single point in Fig. 1.10(b): when we are looking at a problem we begin by asking ourselves if the soil is looser than the critical states. In such states we call the soil 'wet', with the thought that during deformation the effective soil structure will give way and throw some pressure into the pore-water (the

amount will depend on how far the initial state is from the critical state), this positive pore-pressure will cause water to bleed out of the soil, and in remoulding soil in that state our hands would get wet. In contrast, if the soil is denser than the critical states then we call the soil ‘dry’, with the thought that during deformation the effective soil structure will expand (this expansion may be resisted by negative pore-pressures) and the soil would tend to suck up water and dry our hands when we remoulded it.

1.9 Summary

We will be concerned with isotropic mechanical properties of soil-material, particularly remoulded soil which lacks ‘fabric’. We will classify the solids by their mechanical grading. The voids will be saturated with water. The soil-material will possess certain ‘index’ properties which will turn out to be significant because they are related to important soil properties – in particular the plasticity index PI will be related to the constant λ from the second of our critical state equations.

The current state of a body of soil-material will be defined by specific volume v , effective stress (loosely defined in eq. (1.7)), and pore-pressure u_w . We will begin with the problem of the definition of stress in chapter 2. We next consider, in chapter 3, *seepage* of water in steady flow through the voids of a rigid body of soil-material, and then consider unsteady flow out of the voids of a body of soil-material while the volume of voids alters during the transient *consolidation* of the body of soil-material (chapter 4).

With this understanding of the well-known models for soil we will then come, in chapters 5, 6, 7, and 8, to consider some models based on the concept of critical states.

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